On a Simple Model of Nonlocal de Sitter Gravity

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Abstract.

We consider nonlocal modification of the general theory of relativity in framework of the pseudo-Riemannian geometry, with the nonlocality of the form

$$S = \frac{1}{16\pi G} \int d^4x \, \sqrt{-g} \, \big(R - 2\Lambda + \mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R) \, \big) d^4x, \tag{1}$$

where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature R, and $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alambert operator $\Box$.

In the first part of the lecture we give a short overview of nonlocal models based on the action given by (1). In the recent papers we deal with nonlocality of the form $H(R) = G(R) = \sqrt{R - 2\,\Lambda}$, where $\mathcal{F}(\Box)$ is an analytic function of the d'Alembert operator $\Box$ and, after formal generalization also of $\Box^{-1}$. We investigated several classes of scaling factors for flat, open and closed Universe, and we found some new exact cosmological solutions. Specially, we are paid our attention to the scaling factor of the form $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}\, t^2}$. This simple nonlocal de Sitter gravity model, which we denote by $\sqrt{dS}$ gravity, contains an exact vacuum cosmological solution which mimics dark energy and dark matter and is in very good agreement with the standard model of cosmology. This success of $\sqrt{dS}$ gravity motivated us to investigate how it works at the solar system and smaller scales than cosmic scale.

Next part of the lecture deal with our investigations of our model on smaller scales than cosmic scale. The next step is considering corresponding Schwarzschild-de Sitter metric of the $\sqrt{dS}$ gravity model. To obtain an exact solution, it is necessary to solve the corresponding not-trivial nonlinear differential equation. Since this is very hard problem, after the linearization of obtained equation, we found its solutions, which is related to space metric far from the massive body, where gravitational field is weak. The obtained approximative solution is of particular interest for further examining the possible role of non-local de Sitter gravity $\sqrt{dS}$ in describing the effects in galactic dynamics that are usually attributed to dark matter. The solution has been tested on the Milky Way and the spiral galaxy M33 and it is in very good agreement with observational data.

At the end, we extend our model by adding a scalar field into the action, and using corresponding equation of motions we find time-dependant expression for its scalar potential.